

Synchronization and Partial Synchronization of High Dimensional Chaotic Systems

D. DeMaris 11/16/2000

Varieties of synchronization

- At the level of multi-channel spike recordings, we see elevated rates of coincidence described as effective coupling
- Macrostate variable can be average phase relative to offset for mean frequency - resulting in clusters in phase variable

Overview

- Definitions: Oscillations, Coupling, Synchronization
- Maps as model oscillating systems
- Choices in system design parameters
- Five examples of synchronization in cognitive and perceptual processes
- Conclusion: Where/ How to Read Literature

Varieties of synchronization

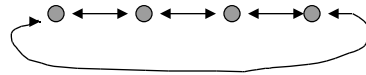
- At the level of local field potentials (Freeman, Bressler) we see apparent oscillation across space - similar time series but different amplitudes
- Macrostate variable is pulse density, sometimes called ensemble average frequency

Maps as prototype chaotic system

Dimension: one macrostate variable standing for average behavior of an ensemble

$$x_{t+1} = f(x_t), t = 0, 1 \dots$$

Higher dimensional systems - N maps with some form of coupling between units.



Synchronization in maps

For high dimensional system (N macrostate variables) coupled in some graph (1 D ring, rectangular lattice, Arbitrary graph), we can speak of synchronization:

Total:
$$\left| x_t^i - x_t^j \right| = 0, t \quad (2)$$

System effectively becomes one dimensional;
also called “coherent”

Partial: Some map units obey (2), others do not; or multiple Groups G_1, G_2, \dots, G_m where (2) is true for units in
System has effective dimension $1 < m < N$

Synchronization in Maps

- Synchronization or Clustering phenomena in coupled maps are interesting because:
 - Only form of equations matters, not “meaning” or interpretation of state variables - theoretical results applicable at various scales
 - Computational structure - effective connectivity of state space “symbols” with minimal changes in network topology, few parameters

Synchronization in Maps

- Clustering phenomena in maps (or other macrostate oscillators) are readily associated with perceptual and cognitive phenomena
 - Segmentation and grouping of objects, parts
 - Formation of hierarchies
 - Flows of attention
 - Formation of representation spaces
 - Organization of spatial fields

Network Parameters for Coupled Map Systems

<u>Input</u>	<u>Coupling</u>	<u>Bifurcation</u>	<u>Readout</u>
1 cell, m cells, all cells	Diffusive/ Laplacian or Difference, Local/Global	Maps of varying topological structure	Instantaneous Distribution
Initial (One Shot)/ Periodic Driven	Fixed/Variable	Fixed/Variable	Time average
	Spatially Homogenous/ Inhomogeneous	Spatially Homogenous/ Inhomogeneous	Spatial Patterns (extracted by Local Mean Field)
State, Bifurcation, Coupling	Input / State Dependent (Hebbian)	Input / State Dependent	Particular unit, Units above (frequency or coherence) threshold

Self-organized hierachical structure

- Ito1. Ito and K. Kaneko, Self organized hierarchical structure in a plastic network of chaotic units, Neural Networks 13 (2000) 275-281.
- Warning: phase used two ways
- Circle map state variable-> phase; input is “phase reset”
- Coupling and bifurcation k give a plot of “phase regimes” (coherent = totally synchronized)

Ito & Kaneko: Self-organized hierachical structure

- Key results:
- Input, while in weakly synchronized phase regime, induces characteristic layered structure for input.
- Units are desynchronized; order apparent only from coupling matrices and structure implied by inter-unit coupling above arbitrary “thresholds”
- Application and readout unclear

Ito & Kaneko

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Yamanoue: Attention with Synchronization

- Yamanoue, Y. Effect of complexity in an oscillatory neural network, *Fuzzy Sets And Systems* (82)2 (1996) pp.253-263
- Cells have tendency for desynchronization; interactions enhance synchronization (coherence), segmenting into groups.
- ONN can focus, defocus, shift attention without additional mechanisms or control

Segmentation with Periodic Coupled Wilson-Cowan

- S. Campbell, S. and Wang, D. Synchronization and Desynchronization in a Network of Locally Coupled (Wilson-Cowan) Oscillators, *IEEE Trans. Neural Networks* 7 (1996) 541-554.
- Wilson-Cowan oscillators in periodic regime
- Local Diffusive Coupling + Hebbian Coupling with “Global Separator”
- Segmentation only demonstrated with well separated objects
- Limited Capacity : 9 objects

Segmentation with Laplacian (Difference) Coupling, Chaotic

- 1. I. Zhao, E.E.N. Macau, and N. Omar, Scene segmentation of the chaotic oscillator network, International Journal of Bifurcations and Chaos 10 (2000) 1697-1708.
- Wilson-Cowan oscillators in chaotic regime
- Laplacian coupling supports segregation into groups - as units become synchronized disappears, they become uncoupled and remain synchronized
- Chaotic oscillation : unlimited object capacity BUT ...
- Complex readout - 3-4 sequential crossings of partition cell (poincare section) with tagging to identify which oscillators identify a group

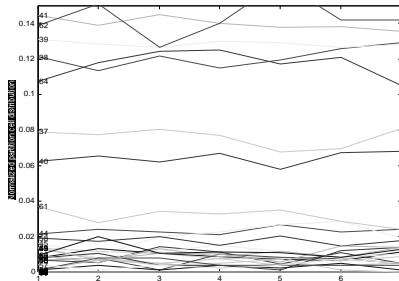
Synchronization Opponent System for Object Recognition

- DeMaris, D. Soca Networks: Computing similarity with nonlinear transients in coupled map lattices. Dissertation, ECE (University of Texas, Austin, 2000)
- Objects presented as synchronized outline on “background rate field” to local diffusively coupled logistic map lattice
- Two Stages (Synchronization Opponents) - Desynchronizing, Synchronizing
- Sample States after two stages - partition cells form a representation space
- Evolutionary Search used to learn “normalized” object

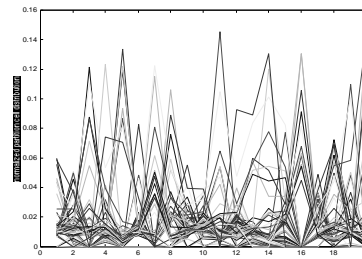
Stimulus equivalence for objects

Sample during synchronization convergence cycle

- Normalization of 2D projections of 3D object - find dynamical parameters which produce most similar distribution across views of each object, using genetic search.
- Minimize collisions (different objects mapping to similar views) by maximum cross-entropy during learning and limits on synchronization.



Distribution of states across 7 views of object 5.5 for best parameters



Mean Distribution of each bin for all objects

DeMaris Soca Network for shape representation

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Summary: synchronization in coupled oscillators or maps

- When reading literature:
 - Look for coupling types
 - Look for number of iterations
 - Thousand of iterations may be hard to justify as biological system
 - Look for readout strategy, biological realism
 - Global or long range coupling from desynchronized state can synchronize rapidly under strong coupling, which can be removed to maintain cluster state

Journals for synchronization modeling work

- Physica D
- IEEE Trans. Neural Networks
- IEEE J. Circuits and Systems
- Int. J. Bifurcations and Chaos
- Neural Networks
- Neural Computing