Chapter 4: Dynamics of Spatially Extended Nonlinear Systems

As background for the experimental work in the next chapter, the fundamental concepts of nonli

near maps and chaotic systems are now introduced, followed by extensions to high dimensional systems, and particularly to uniform spatially extended systems[18]. The aim is not to rigorously develop the material, but to survey the field in enough breadth and depth so that the meaning of the networks and algorithms treated in this study becomes transparent. Many texts providing more depth in low dimensional dynamical systems exist; Ingraham (Ingraham 1991) is a concise survey. To date, no comparable surveys of high dimensional or spatially extended systems exist; furthermore, terminology varies considerably across different disciplines. A special issue of the journal Chaos with an introduction by Kaneko is one possible point of entry (Kaneko 1993), along with a collection of papers on various applications (Kaneko 1993).

There are two approaches to problem solving using complex dynamics. Given the form of a desired outcome, search methods in the control parameter space can be used without requiring deep understanding of the underlying dynamics. The treatment of dynamics, then, is to provide the rational for the search methods and constraints on search developed here.

The other approach involves analytical treatment of the problem so that exact solutions, or at least bounds on the state space are obtained. At the time this project was initiated, the prospects for analytical solutions to problems where the desired outcome involved state distributions on sets of coupled oscillators seemed remote. Accordingly, I hypothesized that search methods might prove effective, even in the absence of a strong theory on bounds of the technique or direct solution methods; the results presented in the next chapter support this strategy. However, techniques have emerged which might lead to more direct solutions than the search methods used here. I will mention relevant mathematical approaches briefly in this review so that it is a useful overview of the evolving state of the art.

In the following discussion, many important terms from the literature of dynamical systems are introduced. For the benefit of readers encountering this material for the first time, these are highlighted in bold type.

ONE DIMENSIONAL NONLINEAR MAPS: DEFINITIONS AND TERMINOLOGY

A map is an iterated difference equation

$$S_{t+1} = f(S_t)$$

where S is a real valued state , f is some function mapping S within a subset of the real number domain $\boldsymbol{R}$, and t is a discrete time step. Iteration implies that the result of

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[18] The term complex systems, to the extent that it is standardized, refers to spatially extended systems.

applying the function at time t is fed back into the computation at to produce the result at time $t+1$. The sequence of states $S^1, S^2, \ldots S^T$ by iteration for t=1,2,…T , is the image of the map. The sequence of states preceding any state S^I are the pre-image of the state. The terms trajectories and orbits also appear in the literature for the sequence of states, with trajectories used for continuous systems and orbits normally used for discrete iterations.

Nonlinear maps use some nonlinear function[19] f, resulting in diverse types of asymptotic behaviors; these asymptotic behaviors are reached after a *transient regime* of variable number of iterates. The duration of this transient depends on the exact initial conditions as well as the exact function and parameters. This variety in transient length and complex structured of the trajectories approaching a stable state, supports the algorithms for forming representation spaces and performing pattern recognition tasks. I will explore this structure by some simple parametric studies in the next chapter.

An **attractor** of a map is the asymptotic state sequence after many iterations, if such an asymptotic state exists. The term attracting set or limit set is also used. The **basin** of an attractor is the set of all pre-image states which converge to the attractor, after some number of iterations.

One crucial distinction for a system is whether for a particular fixed control parameter, different inputs converge to a single attractor or to one of multiple attractors. For the logistic map used here, a single attractor exists for all input states, but the basin structure and transient sequences leading to the attractor are highly variable depending on the particular instantiation of the map (i.e. the exact value of the chosen control parameters).

A dynamical system with multiple coexisting attractors can be used as a model for perceptual and memory processes. Training a supervised neural network consists of shaping the dynamics evolution of a network through its parameters such that the attractor basins map input states into the categories (attractors) desired. This basin structure can be considered as an intrinsic categorization by partitioning the input states into categories corresponding to the attractors.

A well studied map used as a network node (cell, neuron unit, site) in the models described later in this chapter is the asymmetric logistic map:

The equation for the asymmetric logistic map is

$$S_{t+1} = 1 - bS_t^2 \ , \ \begin{cases} -1.0 < S < 1.0 \\ 0.0 < b < 2.0 \end{cases}$$

where b is a bifurcation parameter; changing this parameter forces a structured transition between phases following the sequence of attractor types, which are introduced below:

fixed point $\rightarrow$ limit cycle cascade of increasing period and instability $\rightarrow$ intermittency $\rightarrow$ chaos $\rightarrow$ {limit cycle cascade $\rightarrow$chaos} …

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[19] A nonlinear function is one for which the solutions are not subject to the principle of superposition, i.e., the solutions do not add linearly to generate a new solution .

The changes in attractor type occur abruptly, even with smooth changes in the parameter. The brackets and ellipses indicate that beyond the transition to chaos, there are windows of periodic behavior in the bifurcation parameter values surrounded by regions of chaos, and this repeats infinitely.

A **fixed point** attractor is one for which every initial state leads to the same value. A **limit cycle** attractor is a repeating state sequence of period P; all initial states lead to the sequence, though the **phase** of the sequence (relative to time t modulo P) may vary. A limit cycle attractor may also be referred to as a **periodic attractor**.

This state sequence is the simplest form of **oscillation**. In continuous systems theory and circuit analysis, the conventional meaning of oscillation is a limit cycle or periodic oscillation. In nonlinear dynamics, more complex aperiodic motion is also referred to as oscillatory, which may be a source of confusion in discussions between neuroscientists and nonlinear dynamics investigators.

A **chaotic** attractor is an aperiodic orbit which exhibits sensitive dependence on initial conditions. A system can be more or less chaotic, essentially a measure of how rapidly nearby initial conditions diverge. **Lyapunov exponents** can be computed for a system as a measure of nonlinearity. Since the rate of divergence varies over the set of initial conditions, a system is commonly characterized by the **largest** Lyapunov exponent over the full range of possible initial conditions.

Typically the transition points between these phase regimes are visualized by bifurcation trees for systems with one bifurcation parameter, or phase space plots for **coupled systems** with multiple parameters governing transitions between regimes.

Time series (orbit) plots under various initial conditions, the bifurcation tree for the map, and an example phase space plot are shown in the following figures.

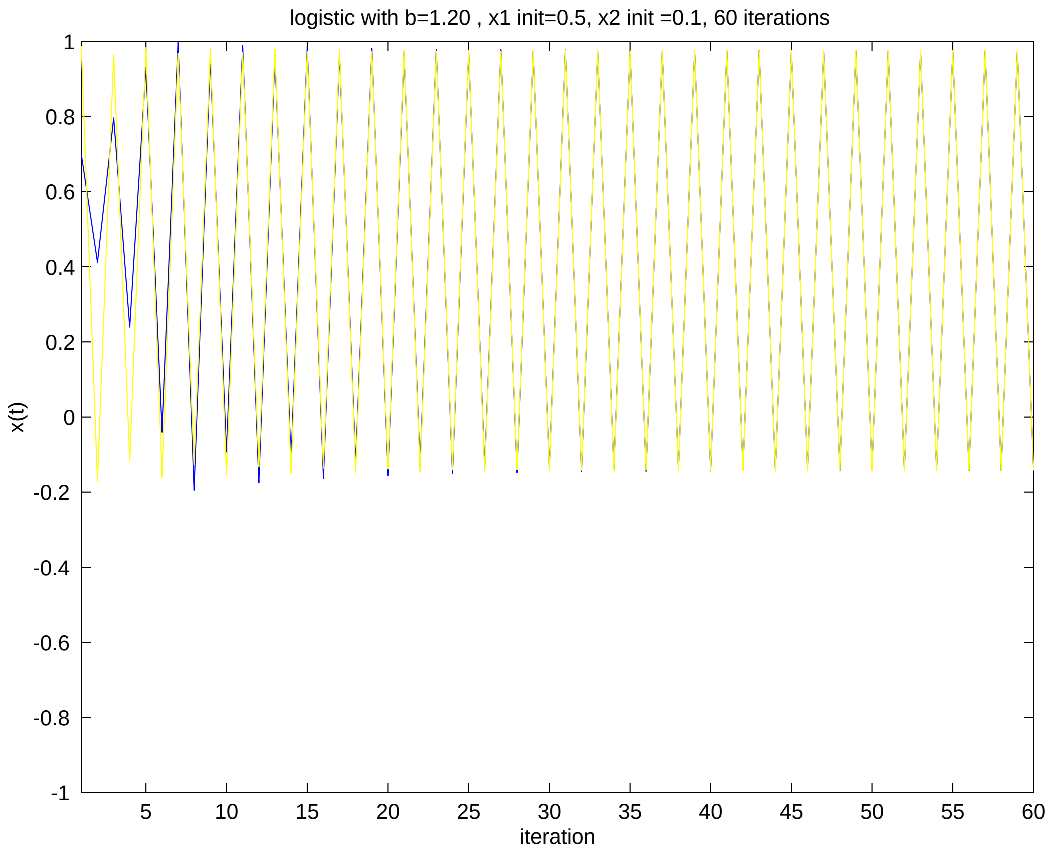

Fig. 14. Time series or *orbit* of the map $x_{t+1} = 1 - bx_t^2$ with x =0.5 and x =0.1 initial states overlaid. The bifurcation parameter b set to 1.2, leading to a stable period 2 attractor for any initial condition.

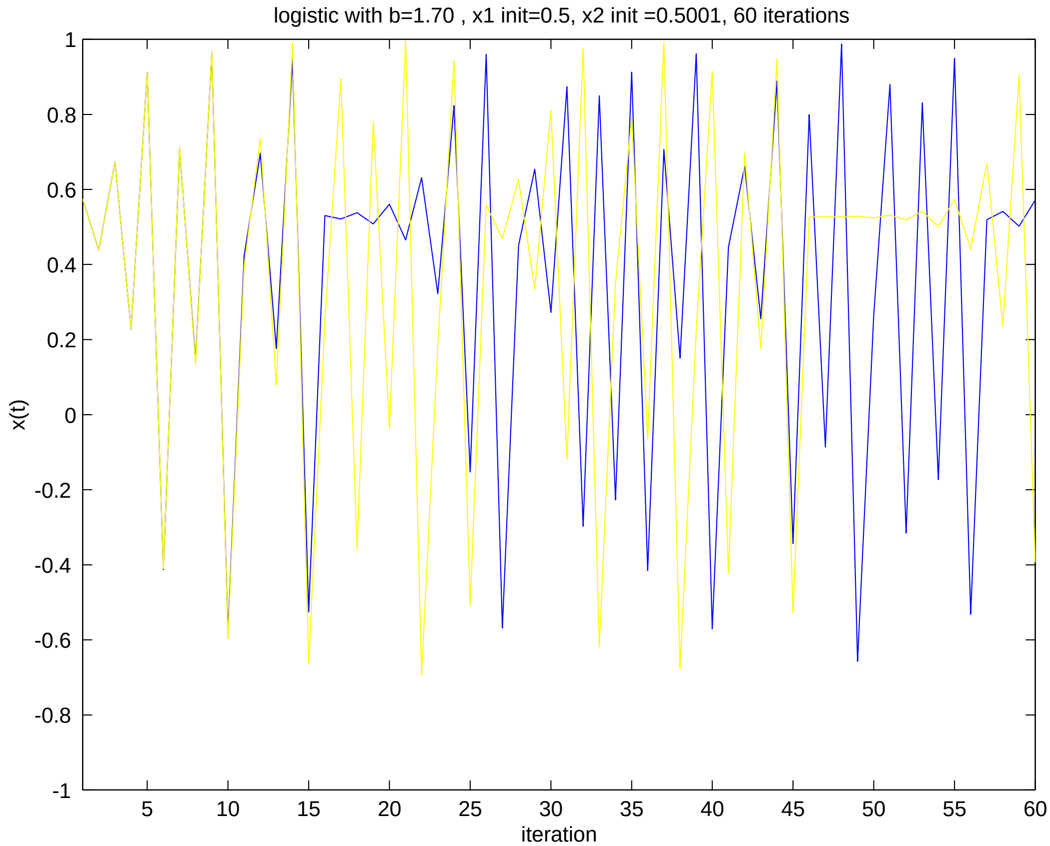

Fig. 15. Time series of initial states x1=0.5, x2=0.5001 superimposed with bifurcation parameter b=1.7, beyond the transition to chaos at b=1.544. The separation of initial conditions differing by .001 illustrates the phenomena of divergence of orbits of nearby initial conditions. The Lyapunov exponent is a measure of divergence (positive exponent) or convergence (negative); the plots here indicate divergent dynamics, while the previous figure is convergent.

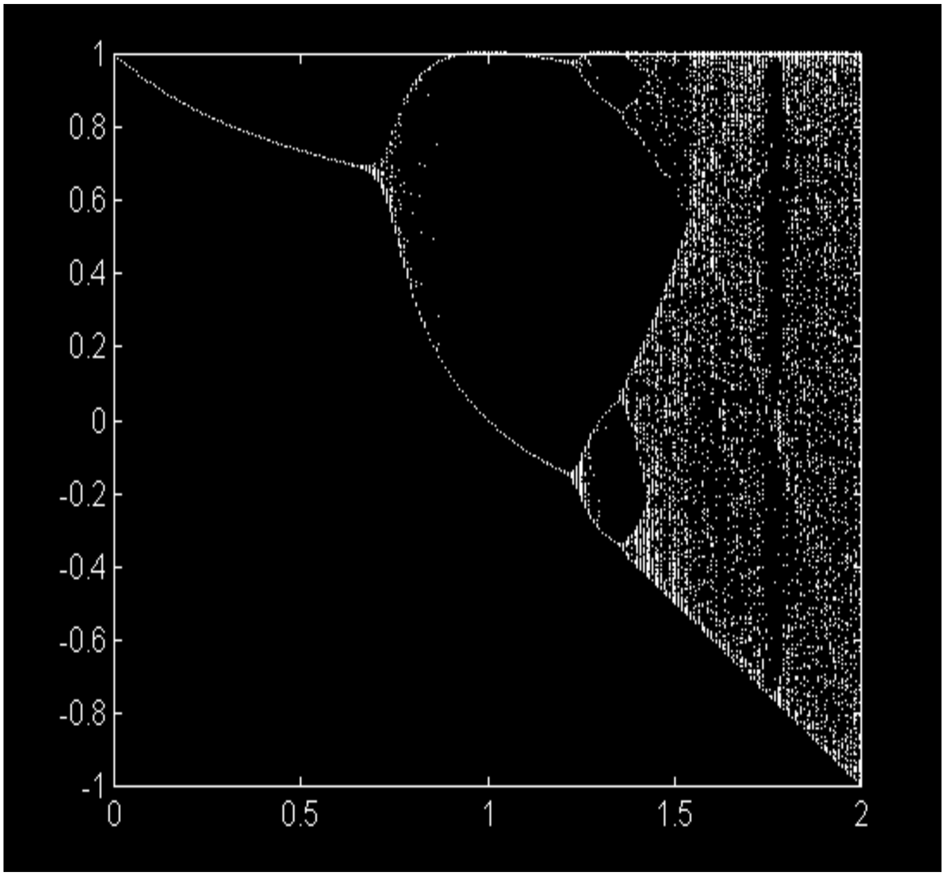

Fig. 16. Bifurcation tree showing asymptotic states of attractors as the parameter *b* is increased. 512 random initial points are chosen for each **b** value and the logistic map is iterated for 100 time steps before plotting. Where multiple state s points exist for the given *b*, a periodic, noisy (unstable) periodic or chaotic attractor is present; the actual state values are cycling between these y axis points as shown in the time series.

Depending on the value of the bifurcation control parameter *b*, the attractor state sequence may be a single state (fixed point), periodic oscillation between a few states (limit cycle), or a pseudo-random visitation of the state space points but within a bounded area (strange attractor, chaotic attractor). Each of the attractor types can be considered as a **phase** or **phase regime** of the dynamics, analogous to thermodynamic phase in classical physical systems. These phase regimes are bounded by **critical values** of the control parameters. When a control parameter is modulated to cross a point where attractors appear or disappear, and in particular change their **topological structure**, the crossing event is known as **bifurcation**. Bifurcations between qualitatively different regions of phase space, such as crossing the transition from limit cycles to chaotic behavior, are termed **phase transitions**.

Bifurcations occur as stability is lost for the attractor. This manifests in slow convergence time, and an increasing number of trajectories which lead away from the attractor. As the bifurcation parameter is increased into the chaotic regime, the unstable periodic orbits remain a controlling influence on the dynamics, effectively forming a "skeleton" for the dynamics.

The evolution or motion of a chaotic attractor in one dimension can be understood as cycling between sets of **unstable periodic orbits** (UPO); the emerging theory of control in low dimensional chaotic systems depends on analytically identifying such UPOs, and applying perturbations to the system to suppress chaos (Barreto, Kostelich et al. 1995). The persistence of such UPOs in chaotic behavior may have implications for the probability of reaching a particular state during the transient evolution of a system, or on the temporal statistics of a time series. This area of dynamics, particularly regarding transients, is not well characterized at the time of writing.

In non-biological systems, bifurcation parameters are typically constant or slowly changing with respect to the equations of motion. It is possible, and assumed by many researchers, that rapid bifurcation is a key aspect of the performance of biological systems.

SPACES, DIMENSIONS, MAPPINGS

At this point in the discussion, we must revisit the notions of space and dimension which have already been introduced, albeit in the context of cognitive theories of similarity as a space of features. Since I will return to that idea, but must use the term space in the dynamics context, the distinction should be made clear.

In the definition of a map given above, I emphasized the discrete nature of the process by using the term state. However, much of the theory of nonlinear dynamics – and more generally topology, of which it is a branch – is formulated in terms of continuous **spaces.** Indeed, the underlying space must be metric by the same criteria described earlier. When referring to the evolution of dynamic variables in $\Re^{n;}$ the dynamics literature normally uses **phase space**, indicating the space of the mapping dynamics.

Space is also encountered in the context of spatially extend systems or networks – here, it has essentially its commonplace meaning, with oscillating computational units or cells located in $\Re^1$ (a line or ring) or $\Re^2$ (a lattice or torus). Networks can also simply be defined on an arbitrary topology or graph, without reference to any embedding in real space.

The term **dimension** must also be revisited. In the context of metric spaces, dimensions typically refer to some measurement or feature, with objects represented as a point (or perhaps as a subspace) in the space. For hidden layer neural network representation spaces, the dimensions may be less directly related to the input; recall that the output units of each RBF classifier in the Chorus of Prototypes system correspond to a dimension in the space of prototypes. In Edelman's terminology, these can be distinguished as proximal (representation) and distal (feature) spaces. In dynamics and oscillatory neural network theory, dimension typically refers to the number of state variables in a system of coupled equations, such as the number of units in a **field** of identical connected units in a spatially extended system. There are additional **measured dimensions** of the orbit itself, characterizing the dynamics and information flow (Grassberger 1991).

We will also be interested in synchronization of oscillating units. In this case, there may be a **structural network dimension** (the number of units) and an **effective dimension**. When a network of coupled oscillators is fully synchronized, all units evolve in parallel and thus behave *effectively* as a one dimensional system.

In the system described here, all of these senses of dimension must come up at least briefly. In equations, I will follow the following notational convention: the variable N will be used for the network dimension, while the variable k will refer to the dimensionality of the representation space. Effective dimensions will be mentioned but will not require standard notation.

HIGHER DIMENSIONAL SYSTEMS: SPATIOTEMPORAL CHAOS

The logistic map introduced above is a typical discrete time nonlinear dynamical system with a single state variable. However, this formalism can be extended to networks of coupled nodes (cells, units) where each node has a real valued state. Such networks are known as **coupled maps.** When coupled maps are arranged in a regular spatial array, the term seen most often in current literature is coupled map lattice, introduced by Kaneko (Kaneko 1989). Other investigators have referred to similar structured spatial systems of nonlinear elements as cellular neural networks[20] (Chua and Yang 1988), fractal chaos networks (Perez and Massotte 1987), cellular dynamical systems (Abraham, Corliss et al. 1991). Due to the iterative or feedback network topology on each node, these systems may be considered as **recurrent neural networks with non-monotonic or bifurcating units**, and some investigators have described work in those terms (Farhat and del Moral Hernandez 1996). Coupled map models with local unit dynamics at the transition to chaos (known as the Feigenbaum accumulation point) have been designated as Feigenbaum networks (Carvalho, R. et al. 1999), and were investigated for utility in pattern recognition tasks by the present author under the name chaotic reaction diffusion networks (DeMaris 1995). Related systems with discrete state values, discrete time, and typically boolean mappings (transition functions) are known as *cellular automata*; many conceptual tools applicable to coupled map systems have been addressed in the cellular automata literature (Wolfram 1986). Some investigators have directly analyzed transformations between the two system types (Chate and Manneville 1989).

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[20] Cellular neural networks (CNN) have been defined in a more general way to encompass both continuous and discrete time systems; the term is more likely to appear in engineering literature (especially circuit theory). For discrete CNN and CML I find no clear distinction apart from terminology.

For higher-dimensional and coupled map systems, the network state is a vector of the states of the constituent nodes. The network attractor, if one can be said to exist by virtue of sufficient coupling, is a **sequence** or **image** of this vector. For spatially structured dynamical networks, a spatial pattern formation behavior[21] at the level of the entire network is evident, emerging from the cooperative and competitive interactions between the patterns and dynamics in the coupled nodes.

This network-level pattern formation may be tuned by controlling the phase regime of the individual nodes, the number of connections between nodes (neighborhood size), the ratio of excitatory to inhibitory connections (for suitable activation functions), or the coupling strength between nodes.

The original papers on spatial pattern formation in locally coupled map lattices (Kaneko 1989); (Kaneko and Tsuda 1994) introduced many visualization techniques and correlation measures to characterize the rich behavior in various parameter regimes. In general, the long time behavior investigated by Kaneko is not applicable to the system described here, and as argued in the neuroscience review, is probably not applicable to rapidly developing perceptual processes. The present work investigates and uses only brief transients (10-16 iterations), while Kaneko's original simulations of locally coupled maps examine the dynamics after 10,000 steps, omitting any consideration of the transients.

The term attractor is sometimes used at the network level, but is generally less useful for spatial lattice systems with weak or local coupling, where oscillations and competition between **clusters** (oscillation modes, sites in the same state, attractor, or basin) form **dynamic patterns**. The **evolution** of a network from an initial state under relatively low coupling results in an organization in which patterns of continuing activity between interacting cells are spatially bounded by "frozen" areas, in which the neighborhood interactions reach a stable state. The local active areas are referred to as **domains**, while the frozen separating regions are domain boundaries. In a sense, the network organizes itself into sub-networks, with the activity pattern in a domain more conventionally related to the definition of an attractor.[22]

Systems with strong random coupling or global coupling, in contrast, can be shown to reach an attractor which may be equivalent to a one dimensional map; this **synchronization** process is taken up later. **Intermittency**, or **chaotic itinerancy**, is a phenomenon appearing in a small, weakly chaotic region of the parameter space, in which the dynamic behavior is a blending or linking of unstable periodic attractors existing in isolation in the more ordered regions of parameter space. Over time, individual periodic attractors are separated by sequences of intermittent chaotic transitions. This particular dynamics has been proposed by Tsuda as a supporting mechanism for binding (Tsuda 1992) and may offer advantages as an associative memory

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[21] Kaneko uses the term spatial bifurcations for formation of separated islands or domains; I would prefer to reserve that term for dynamical scenarios in which spatial patterns actually influence local or global bifurcation parameters.

[22] The concept of a domain as used by Kaneko applies only when the lattice has been iterated for many (> 10,000) generations, so that transients have died out.

which overcomes the limitations of previous parallel distributed processing networks with respect to the issue of **compositionality** (the recovery of bound or integrated features in a composite memory).

Various network or neighborhood topologies have been reported in the literature on spatio-temporal chaotic systems. Network nodes may be **locally coupled** to adjacent nodes, **diffusively coupled** to a small region of the lattice with connection strength weighted by distance, **globally coupled** to every node, **randomly coupled** to a non-spatially localized set of neighbors, or some blending of these conditions. The coupling may take any functional form, including diffusive (multiplicative), difference or Laplacian coupling, or time varying functions. Coupling may be **symmetric** or **asymmetric** between adjacent units; it may be **homogeneous** over the spatial extent of a lattice, or **inhomogeneous**. The terms uniform and regular have also been used to denote homogeneous bifurcation and/or coupling.

Coupled Map Lattices

A coupled map lattice (CML) is a dynamical system with discrete time, an extended field of state variables in discrete space, and continuous state. Of course, we approximate continuous states with floating point values in map computations, so strictly speaking chaotic attractors must all in reality be periodic with very high period. Kaneko (Kaneko 1993) describes the generic CML modeling process for a physical system as follows:

1. Choose a set of field variables on a lattice. Typically these variables represent macroscopic (distributed) qualities, such as temperature, fluid velocity field, local concentration of a chemical substance, or in our case neuron pulse density in a local population.
2. Decompose a process into independent units, such as convection, reaction, diffusion, etc.
3. Replace each unit by the simplest possible parallel dynamics on a lattice, consisting of a transformation function at each lattice point or a coupling term among suitably chosen neighbors.
4. Carry out each process successively. In the present model, this means that at each iteration, a diffusion step is performed, then a reaction step.

The logistic map was originally chosen for its well understood properties rather than any explicit biological motivation[23] for that *particular* nonlinear function. The equation is known to be numerically stable when the state (and perturbations or forcings) are maintained within certain bounds, and the critical points where bifurcations occur are known. The behavior of logistic maps in a toroidal lattice with both local and global diffusive couplings has been extensively studied by Kaneko. These studies investigated

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[23] However, the range of the asymmetric logistic map [-1 to +1] could be appropriate for modeling a process of *modulation about a background average frequency*. Some investigators of IT cortex present evidence that modulations of the background rate in individual units (Richmond et al.) or populations (Gochin et al.) predict the stimulus present. The zero value would be the background rate.

random initial conditions at each site. The pilot study leading to this dissertation (DeMaris 1995) was possibly the first to explore the dynamics and resulting distributions of a diffusively coupled logistic lattice *with structured spatial data* supplied as initial conditions.

The specific steps used for image processing in the system developed here are as follows. Note that the complete sequence of steps here is one iteration imbedded in one stage of the larger computation.

DIFFUSIVE COUPLING STEP

$$S_d(x,y) = (1-c)S_t(x,y) +$$
$$\frac{c}{4}\left[S_t(x,y+1) + S_t(x,y-1) + S_t(x+1,y) + S_t(x-1,y)\right]$$

where: d is the intermediate diffusion array, t is the current time step, x, y are the spatial indices of the pixel array S at the center of the diffusion neighborhood, S is the state variable at each site of the shape array, and c is the coupling constant restricted to the range <0.0 to 1.0>.

The diffusion or averaging step is implemented as an 2-D filter with the convolution kernel:

$$\begin{bmatrix} 0 & c/4 & 0 \\ c/4 & 0 & c/4 \\ 0 & c/4 & 0 \end{bmatrix}$$

where c is the coupling constant .

Diffusion is followed by a squashing step, which insures that the state remains bounded in the stable domain of the reaction step as described below:

$$S_t(x,y) = (1-c)S_t(x,y) + d_t(x,y)$$

LOGISTIC MAP STEP

The second computational unit applied in each time step is the logistic map:

$$S_{t+1}(x,y) = 1 - b(S_t(x,y))^2$$

where *S, t, x, and y* are as stated above and where:*b* is the bifurcation parameter, restricted to the open interval (0.0,2.0).S is restricted to the open interval (-1.0, 1.0).

The dynamics of the logistic equation are such that given any initial state, after some transient number of iterations the system will reach a steady state attractor which is fixed (low b), periodic with increasing cycle length and transient length, or chaotic (higher b values). As long as the initial input states S representing an image are bounded as described above the system will be numerically stable. In the periodic regime, the iterations required to converge to the attractor vary from 1 to 100 or more; longer convergence times are observed in the vicinity of the critical (bifurcation) points. This phenomena can be seen in diagrams in the next chapter illustrating the evolutions of transients as the bifurcation parameter b is scaled.

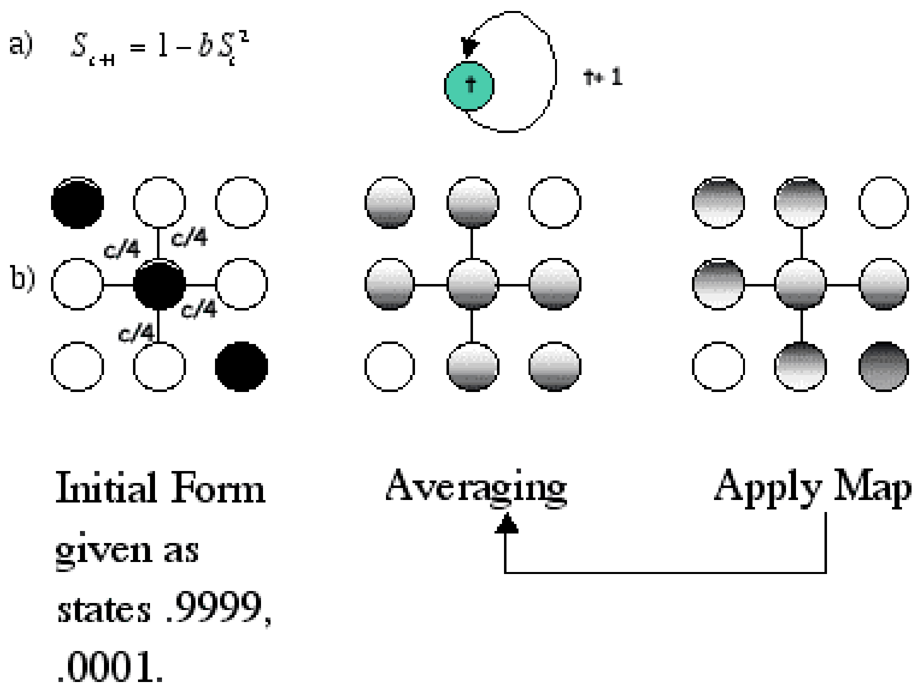

Fig. 17. A pictorial illustration of the CML architecture. The logistic equation is executed in each node of the CML. Each node implements an *iterative* or *recurrent* computation, illustrated by the loop on the node illustrated. b) A regular spatial lattice of such nodes is connected to nearest neighbors, with the NSEW edges labeled averaging a scaled fraction of the state value of their neighbors at each iteration of the lattice. The illustration is intended to show how a contour of black "zeros" in a background of white "ones" is diffused by the averaging process, then transformed by the map. The sequential application of both operations is one iteration of the coupled map process.

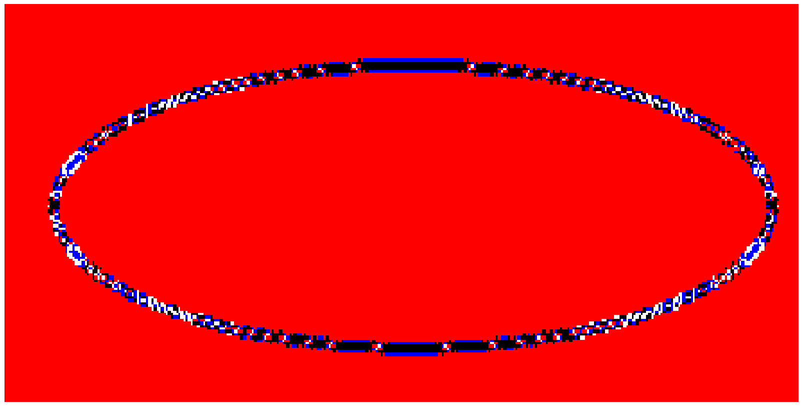

Fig. 18. A snapshot in time of an evolving coupled map lattice. An ellipse form with values .0001 on a background field of .9999 given as an initial state is evolved through the diffusively coupled CML process described above; a cyclic color map highlights distinctions between the state values in the interval [-1,1]. The thickness of the ellipse indicates the propagation of the diffusion wavefront, while the local spatial structures show the effects of coupling and map operators to produce characteristic local statistics for a particular curvature region.

Synchronization in Coupled Map Lattices

Several types of synchronization may occur in CML systems, given the different phase regimes that the individual component maps and collective can operate in. If individual units reach fixed points, the only possibility is totally synchronized, possibly in different domains or clusters.

If individual units exhibit periodic oscillation, due to their local bifurcation parameters or strong coupling, they may exhibit phase clustering, with P clusters corresponding to the possible phase offsets in limit cycles of length P relative to time t modulo P. If all units are in the same phase, they are defined as totally synchronized.

If individual units exhibit more complex (chaotic or itinerate) behavior, they may be still be totally synchronized. Apart from such total synchronization, a variety of synchronization types and clustering identification techniques are beginning to be noted by investigators.

Partial synchronization is defined (Maistrenko, Popovych et al. 2000) in contrast to total synchronization. For a set of units coupled in some graph, total synchronization is defined as the case in which

$\left|s_t^i - s_t^j\right| \to 0, t \to \infty$ The system of N units (N dimensional network) effectively operates as a one dimensional map when synchronized.

Partial synchronization, then is the situation where some units obey the synchronization condition above while others do not. The network may be characterized by clusters C1, C2, …Cn < N, where N is the number of units. These may be spatially segregated into contiguous oscillating *domains*.

For Maistrenko and colleagues, the emphasis is on clustering of units which reach an asymptotic state sequence which falls short of full synchronization. In contrast, another type of partial synchronization will be the focus of the rest of this thesis. Over the time course of a dynamical evolution from an inhomogeneous initial state towards synchronization, in either the full or partial sense of Maistrenko, we can consider the distribution during the transient state as exhibiting partial synchronization.

If such partial synchronization is measured by sampling the dynamics (or simply halting them, if such control is available), the differing rates of convergence of local configurations to the asymptotic distribution can be used in information processing.

Clustering Phenomena in Globally Coupled Chaotic Maps

The dynamics of a coupled, spatially extended system of maps can no longer be visualized as a bifurcation tree (the pitchfork diagram show above); instead, a phase regime plane is used, where each point corresponds to a family of attractors of the same type or mixtures of attractor types, and bounded regions correspond to phase regimes in the entire network state space. In the more complex regimes described later, mixtures of the simple attractor types (such as cycles and intermittency) may co-exist in physical space, or move in the physical space of the network as traveling waves.

A schematic plot of the control space for a network of **globally coupled** logistic map nodes, where each node is coupled to a mean field or average of all nodes at each time step, is shown below. The axes of the plot correspond to local bifurcation parameters, and coupling parameters between nodes. While the boundaries between the network phase regimes are simple in this depiction, they can normally be very complex and intermingled even for single map units, (e.g., the appearance of a period 3 window surrounded by chaos can be seen in the bifurcation tree).

The structure of bifurcations is known as the **route to chaos** of a chaotic nonlinear system. The individual logistic map at each site, with no coupling, cycles through a **period doubling limit cycle cascade,** reaching the chaotic regimes at a critical value *b*. The addition of spatial arrangemetnts and coupling to the low dimensional dynamics picture complicates the description of dynamical structure. The intermediate regimes for locally coupled maps are considered to have "frozen random" pattern selection behavior in which **domain boundaries** form. Higher couplings produce larger domains and ultimately a pattern formation behavior.

The globally coupled map has a toroidal collapse route to chaos. In this route, the boundaries between the network phase regimes are monotonic with respect to the control or parameter space. In globally coupled maps, domains are more unstable and clustering is the dominant phenomenon.

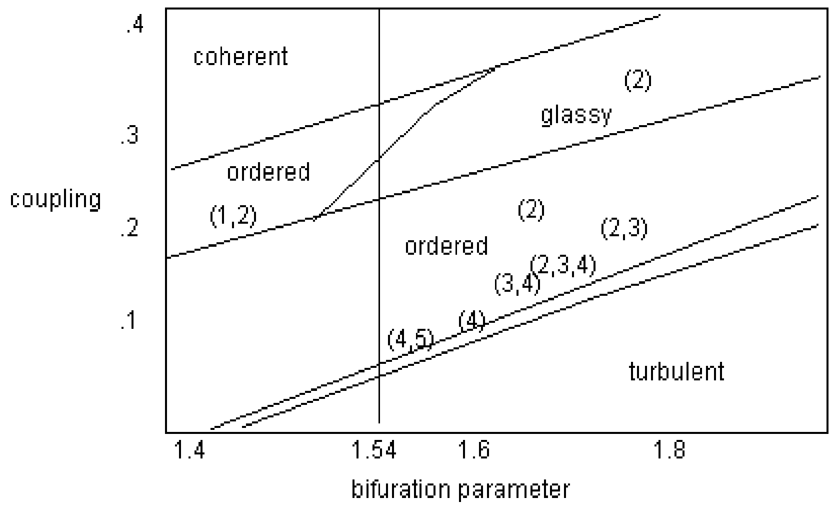

Fig. 19. The control plane and resulting *phase regimes* for a globally coupled map lattice, after (Kaneko 1990). The line at 1.54 is the critical point (transition to chaos) for an uncoupled map. The coherent or synchronized regime indicates that all sites exhibit the same orbit. The numbers in the ordered and glassy regimes refer to the number of dominant clusters, defined as those with basin volume more than 10%. A cluster is a set of lattice sites oscillating in the same attractor, though not necessarily with the same phase. Note that the even when individual sites would be chaotic, strong coupling can enforce coherence and complex behavior.

THE SOCA DESYNCHRONIZATION-SYNCHRONIZATION CYCLE: A TIME-VARYING CML

Having assembled the conceptual tools underlying the network at the heart of the hybrid pattern recognition system, I now describe the small extension of the classical coupled map lattice. This extension, motivated by the experimental observations of changing slow wave potentials and correlations (Bressler 1995), is shown in the next chapter to increase recognition performance (both the recognition rate and average recognition time) achieved in bounded iteration counts for a recurrent network. This key extension is to use time-varying parameters, alternating between opposing epochs of desychronization and synchronization.

Once again, the reader's attention must be called on to note a subtle shift in th meaning of synchronization, as it is used in a graph theoretical and statistical perspective on dynamics; this will be explained in more detail in the following section. In this graph theoretical perspective, desynchronization (broadening of the distributions) by loosely coupled chaotic dynamics is followed by partial synchronization with more strongly coupled chaotic dynamics.

The two meanings of synchronization are closely related in a high dimensional system. In my use of coupled maps for shape representation, the initial conditions are confined to a small subspace of the phase space; this is synchronization in the new sense. The chaotic dynamics, even in a strongly coupled system, will desynchronize in the sense that for some transient period a range of time series will emerge through the neighboorhood actions, whose correlations depend on the initial conditions but also diverge in a pattern dependent fashion. When time series synchronization occurs – either through temporary fluctuations in the transients, or through the gradual formation of synchronized domain strutures – we will see the concentration of the system state in a subspace again. This concentration may be apparent in instantaneous measurement of the system state, or by examining time averaged occupancies of subspaces.

These operations take place in a homogeneous, orientation-sensitive array of recurrent logistics maps. By homogeneous, I mean that the bifurcation and coupling parameters are uniform across the entire array. By orientation-sensitive, I refer to the spatial asymmetry in the coupling kernel (i.e. NSEW neighbors coupled, diagonal neighbors uncoupled).

Expressed in algorithmic form, the procedure is:

```
procedure synchronizationOpponentNetwork
  image = threshold(downSample(readImage)))
  for iterations = 1 to t1
     diffuseImage = filter2D(couplingMatrix,image)
     image = logisticMap(diffuseImage,b1,c1);
  end // Desynchronize stage
  for iterations = 1 to t2
      diffuseImage = filter2D(CouplingMatrix,image)
      image = logisticMap(diffuseImage,b2,c2);
  end // Partial Synchronize stage
end  // procedure
```

Because of the variation in divergence and convergence times, a specific set of bifurcation, coupling, and iteration time parameters {b1, c1, t1, b2, c2, t2} has a characteristic response to any given image or family of images. Each image can be considered as a set of overlapping initial configurations of size t1+t2; by the end of the Soca process above, information about local configurations from a window of size t1+t2 is contained in each unit (pixel in the processing array). The set of initial configurations comprising one image may be highly synchronizing for those parameters, while another image may be less so less so.

The intuition behind the network operation is that images in some category are considered as *productions* of a *stochastic language* on an alphabet α whose symbols are local pixel configurations. We seek parameters for the first (desynchronizing) stage which, for this language, have the appropriate divergence rate matching parameter determined characteristics of the second (synchronizing) stage. The second stage must have characteristically avoided regions of state space and state transtitions such that images in the category will *converge near a characteristic sparse distribution.*

Similar images should result in similar output distributions, and the inherent characteristics of coupled chaotic systems – divergence of nearby states with time under low coupling, but convergence to synchronized or partially synchronized states with high coupling – offer a potential computational framework.

This computational framework can be viewed in terms of dynamical recognizers reviewed earlier. Alternatively, it can be described as a generalization of a problem in graph theory known as the *road problem;* it also has similarities to Markov chains and probabilistic finite state automata. These descriptive frameworks are briefly described in the next chapter on representation and learning.

This interaction between the specific initial configurations on a shape boundary and the dynamics is an example of *cooperative processing*. Cooperative phenomena, and particularly pattern formation processes, are distinguished by Palm (Palm 1982) by the following criteria:

1. The phenomena arises from the interaction of a large number of similar components.
2. The laws governing the interaction of the components are local.
3. The dynamical laws are the same for all components.
4. The local dynamics should not contain the “symmetries” of the global pattern evolving through the local interactions. It is usually difficult to predict what pattern will arise from the local interactions.

The essence of the activity flow in the present network is cooperative processing mediated by the *opposed processes* of desynchronization and synchronization, hence it is designated as *synchronization opponent cooperative activity*, or **Soca network**[24].

The table below indicates the particular network parameters investigated here, selected from an expanded *network parameter space* of possible single layer CML systems.

Table 3. Network Design Choices for the Soca System

	Input	Coupling	Bifurcation	Readout
Structure	All cells	Local Diffusive	Logistic	partition cell occupancy of all states in diffusion wavefront,
Spatial	Homogeneous	Homogeneous	Homogeneous	All states
Temporal	Initial condition (one shot)	Variable “opponent” stages	Variable (opponent stages)	Instantaneous

The entries in the table above represent parameters of network design using CML derived systems. The set of choices shown are *fixed constraints*, within which an evolutionary search proceeds to discover solutions to an object recognition problem. A more ambitious evolutionary search could choose to optimize networks choosing different alternatives for some or all of those parameters.

MACROSTATE VARIABLES AND MEDIUM SCALE NETWORK MODELS

Before proceeding to experimental methods and data, I now return to the general topic of modeling biological networks with oscillators and synchronization phenomena. The following sections serve two goals. The first is to highlight the history and justification of such mathematical systems as neural models. Second, I mean to survey

[24] The name is inspired in part by the admonition of Walter Freeman that neuroscientists need to learn to dance.

closely related work in networks oriented toward some specific perceptual modeling or pattern recognition function.

As noted earlier, connectionist models have typically assumed rate coding and monotonic activation functions, which correspond closely to the common assumptions of transfer functions for single neurons, and to assumptions about multiplicative weighting as the essential operator controlling network information flow. Some investigators are more cautious, noting that units in connectionist networks may correspond to larger structures. For example, questions raised regarding the reliability of single neurons for rate coding suggest that a sigmoidal activation function unit might be better interpreted as the average behavior of many parallel detectors converging on a family of readout neurons (Softky and Koch 1994).

An alternative to modeling single neurons or putative “parallel averaged networks” is to more directly model large scale dynamics, consisting of thousands of neurons. This has historically been the domain of *statistical mechanical* (Amari 1974) and oscillatory models. One of the first such oscillatory models still in relatively wide use [25] was developed by Wilson and Cowan (Wilson and Cowan 1972); (Cowan 1974). Continuous variables represent activity levels of excitatory and inhibitory sub-populations, rather than activity of single neurons.

There is one crucial difference between the Wilson-Cowan (WC) model and the use of maps to model oscillatory brain dynamics; while both maps and the WC model can generate chaotic time series and be coupled in spatial aggregates, the WC model also has “resting”, non-excited states. Since a map with chaotic control parameters is chaotic for any input, there is no equivalent rest state. It would be possible to introduce additional nonstationarities in the model, with a baseline fixed point attractors (and corresponding bifurcation parameter) designated as a rest state. Input shifts the bifurcation parameter to leave the fixed point state, perhaps with some decay to the resting state. In the modeling here, I address this by simply ignoring “background” states beyond a diffusion wavefront region of interest in the evolving pattern by omitting the highest bin count when gathering the statistics. Given the current algorithmic “back end” recognition process, the use of histogram functions intended for image processing would require similar suppression of the rest state values even if a more complex input coupling and evolution dynamic were used, without really contributing to the essence of the project.

When modeling physical or psychological phenomena with spatially extended (field) dynamics, it is common for each variable in a field (i.e. each unit in a coupled map lattice) to represent a quantity associated with an aggregate of microscopic units. This kind of representation, originating in statistical mechanics or fluid dynamics, is known as a macrostate variable. Temperature or instantaneous velocity of a fluid, for example, are macrostate variables in the study of fluids. In neural modeling, the macrostate variables are quantities like ensemble activation (average spike train frequency of all units), temporal phase distribution, or ensemble average frequency (pulse train density or spikes / unit time measured over the whole ensemble). Parameters in

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[25] Cowan cites earlier work by Buerle (1957) and Griffith (1963) as historically important in this research stream; I have not encountered recent work citing them.

equations may also be considered as macrostate parameters, indirectly capturing quantitative effects of distributed aggregates like neurotransmitter fluxes, and excitatory-inhibitory ratios of neural sub-populations, or distributions of delay times which induce bifurcations in large networks.

Earlier the distinction was made between state variables and control parameters, with the latter consisting of bifurcation and coupling parameters. In large scale models, control parameters - especially bifurcation variables – generally will not have a direct map to single parameters in the underlying micro-circuit model. In fact, bifurcation variables can be interpreted as subsuming the effects of widely separated neural tissue, such as cortical regions and sub-cortical nuclei, or long range connections between cortical regions.

As noted earlier, the trend in interpretation of EEG signals and evoked potential responses is as a signature of large scale coordination and control processes between the regions that brain imaging indicates cooperatively produce computations. The sudden step function change in network parameters above should be interpreted as an example of such large scale control, where staged or periodic volleys from cooperating cortical or subcortical regions effect this rapid change in bifurcation and coupling parameters.

The systems described here should be regarded as spatiotemporal cooperative systems acting on vectors in **pulse density** space, with the step-function changes in bifurcation and coupling parameters representing slower control dynamics implemented by modulation from separate sub-populations. The control dynamics of bifurcation and coupling at the population level supplement the traditional neuron level control dynamics of gating, inhibition, and feature selection. These population level control dynamics may be more easily correlated with MEG and EEG observables than the traditional control dynamics acting at more local scales. In turn, they may produce neuron level observables such as modulations in correlations of neurons engaged in a processing task. I will return to this subject in the final discussion section.

The next section addresses the question of how the use of chaotic maps is justified in terms of standard neuron models, without addressing questions of learning.

SYNCHRONIZATION PHENOMENA AT MULTIPLE SCALES

If a map is to be regarded as representing the aggregate behavior of a large system of neurons, it becomes clear that we must explain oscillatory and synchronization phenomena operating at multiple scales of the brain, from micro-circuits of a few neurons to large networks. This task has been addressed by several investigators. In one such study, Wennekers and Pasemann investigate coupled pools of sigmoidal activation neurons with random diffusive coupling and a probability distribution of coupling strengths. They found that for appropriate parameters the temporal behavior of the whole system can be described by a single, low-dimensional equation (Wennekers and Pasemann 1996). This is proven true asymptotically for long times and system sizes approaching infinity. Even for fairly small networks (N=50 for two interconnected pools) with 50% standard deviation in coupling strengths, a similar bifurcation structure between the low dimensional system and the full network (average activation of all micro-circuit neurons) is seen. The network bifurcation structure has a period doubling cascade leading to chaos, similar to the logistic map used here, but with the possibility of multiple coexisting attractors for any given coupling between pools. Their work situates itself as an extension of seminal work by Anninos, Harth and colleagues (Anninos, Beek et al. 1970); (Harth, Csermely et al. 1970), and Palm (Palm 1982), which demonstrated that *threshold modulation* in random networks leads to mean activity input-output curves described by a single humped function.

In a series of studies, these earlier workers performed discrete time simulations of random networks of threshold neurons of mixed excitatory and inhibitory types. Parametric curves (reproduced below) of activity levels α resulted from studies of the networks which varied the parameters shown on the following page along with a typical activity transfer function.

Network parameters affecting activity curve

h percentage of inhibitory cells

μ^+ average number of synapses of an excitatory cell

μ^- average number of synapses of an inhibitory cell

η threshold of all neurons (number of excitatory synapses to fire)

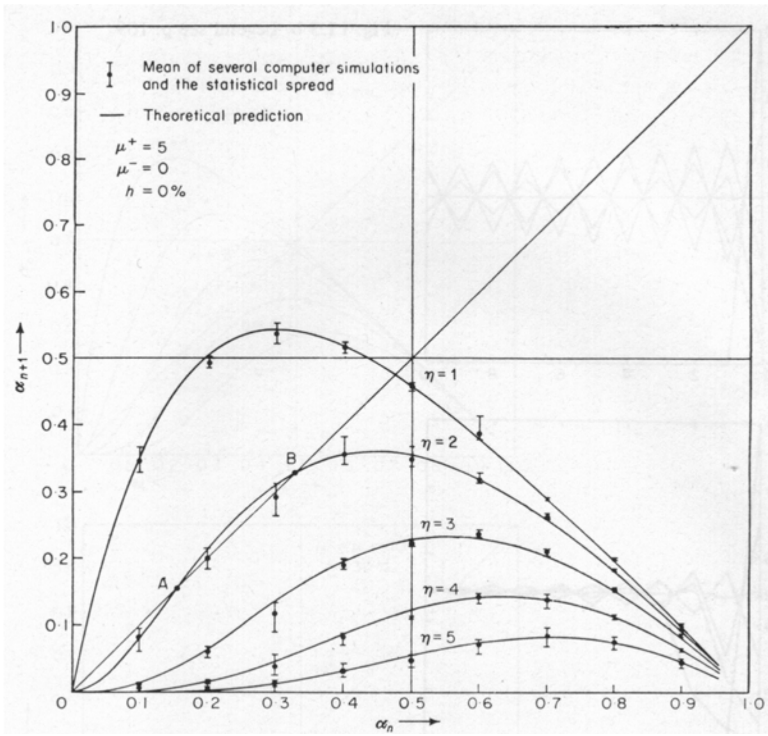

Fig. 20. Activity at time step t+1 vs. t. Note that the form of these curves, are single humped maps – the same input output structure governing the logistic map. This illustrates one scenario to produce the input-output form, characteristic time series and bifurcation structures by micro-circuit models. Changing excitatory-inhibitory ratios and the firing threshold act as the bifurcation parameters of this model. From Anninos, P. A., B. Beek, et al. (1970). "Dynamics of Neural Structures." Journal of Theoretical Biology 26: 121-148. Reproduced with permission of Academic Press.

BIOLOGICALLY PLAUSIBLE COUPLING VALUES

A question that should be raised when thinking about CML systems as models of medium or large scale neural dynamics is what, if any restrictions on the range of coupling parameters should be considered as biologically realistic?

One position would be to consider anatomical data on the proportion of lateral connections to recurrent connections in mini-columns as a bound, resulting in a smaller upper bound (perhaps .03) than is typically found by evolutionary learning in my experimental work. I would argue that such a restriction is unjustified, because the underlying micro-circuit dynamics which regulate synchronization may not require large numbers of independent connections, instead relying on more subtle mechanisms. For example, spike doublet firing mechanisms have been implicated in long range synchronization (Traub, Whittington et al. 1997). Thus the coupling strength should be understood in terms of ratios along the continuum achievable by any such synchronizing mechanisms. hese will of course depend on a non-zero anatomical connection density, but need not bear any linear relationship with the lateral connection density.

REVIEW OF ANALYTIC RESULTS ON SYNCHRONIZATION AND ENSEMBLE DENSITIES OF MAPS

The previous sections have covered the basic definitions needed as a basis for understanding the dynamical network function for representation forming representations and for pattern recognition. I will now briefly describe recent theoretical developments which hold promise for future work on more direct computations of required parameters, or for establishing bounds on the applicability of the technique. The algebraic treatment of graphs, known as spectral methods, has been extensively developed since origninating in 1972 (Donath and Hoffman 1972) for graph partitioning, with application to integrated circuit design. Building on these techniques, Wu has established several theorems on the lower bounds of coupling required for full synchronization on various topologies of coupled map lattices (Wu 1998).

These results apply when the bifurcation and coupling parameters are homogeneous or uniform. Three cases are treated.

For globally coupled logistic maps, coupling to the mean field exceeding a threshold $|1-c| < \frac{1}{b}$ guarantees synchronization.

For coupled maps in which each of N units is symmetrically coupled to k neighbors with coupling $\frac{1}{k}$, the system almost always synchronizes for large k.

For coupled maps with uniform symmetric coupling on a connected graph of n nodes, a region in the (b, c) parameter space results in full synchronization. The region is bounded by the constraints

$|1-cm| < \frac{1}{\sup_x \|Df\|}$, $|1-c\alpha| < \frac{1}{\sup_x \|Df\|}$ where α is the algebraic connectivity of the graph (smallest nonzero eigenvalue of the graph Laplacian matrix, Df is the

Jacobian matrix of the map function *f* maximum divergence over nearby points in the map domain, and m = min(n, $2deg_{max}$) for the graph.

Another result due to Wu is that for uniform discrete time maps with homogeneous *additive* coupling (Wu 1999). For the case in which the connection topology matrix has zero row sums, the dynamics of the coupled system after synchronization is proven equivalent to some parameterization of the *uncoupled* map. This result may have implications for efficient encodings which are generated from a high dimensional system, suggesting that an encoding or matching process could generate the effective dynamics with a one dimensional system. The proof does not hold however, for diffusively coupled maps used in this thesis.

Synchronization per se is not a goal for the following description of a pattern recognition system; on the contrary, in order to form a representation space, complete synchronization must be avoided or the representation space becomes effectively one dimensional, with little separation among object representations likely to occur even if sampled prior to complete synchronization. One possibly use for these theorems would be to set constraints for any learning or search process, since coupling values guaranteed to synchronize may be unsuitable for the purposes of forming representations, if it occurs rapidly.

The Time Course of Evolving Distributions in Ensembles

As noted, most work in dynamical systems focuses on the long time equilibrium behavior of systems, in contrast to their transient behavior. One recent exception is work on the evolution of densities (distributions) in ensembles of identical maps (Driebe 1999).

Driebe's work takes the statistical mechanical viewpoint, studying the distribution of states for an ensemble of identical maps over a distribution of initial conditions. While the density concepts and the methods developed are also applicable to equilibrium states, the work is notable in its emphasis on distributions during the transient evolution toward the limiting equilibrium distribution.

Densities may be measured instantaneously over an ensemble, or over time. Given the latter emphasis, complex motion (orbits) in low-dimensional chaotic systems is naturally described by densities. Density means the occupancy of a region of phase space, i.e. the fraction of the ensemble in a particular subset of the domain of the function. Initial nonequilibrium density represents an ensemble of uncoupled maps with different initial conditions (or perhaps just uncertainty about the initial conditions).

Behavior of orbits in a map and densities in an ensemble of maps may be strikingly different. A typical orbit in a chaotic system looks qualitatively similar forward and backward in time, even if the dynamics is non-invertible. In contrast evolution of density is usually obviously time oriented. Thus density evolution is not reducible to trajectories.

For non-invertible systems with chaotic trajectories, evolution of densities will show regular behavior; for systems with regular (i.e. periodic) trajectories, the density will mirror the orbit level. For chaotic maps, nearby trajectories diverge, while initially different densities, over an ensemble of identical maps converge. Instantaneous densities of low dimensional systems may rapidly (e.g. only a few iterations) approach the equilibrium density.

The evolution of densities is described by the Frobenius-Perron operator. The spectral decomposition of this operator can be used to compute decay rates of correlation modes (peaks in the Fourier transform corresponding to poles in the complex frequency plane). The *spectrum* of an operator is the set (discrete or continuous) of eigenvalues of the operator under consideration acting in a specified functional space, and is different for different functional spaces. For uncoupled maps the distribution of a random ensemble can be computed exactly at each time step.

For the purposes of the discussion here, the main message is the *rapid evolution of distributions* toward an equilibrium value for chaotic dynamics; however, it is not entirely clear if this rapid evolution applies to *coupled* map systems with highly structured distributions. The relatively small number of lattice sites, strong peaks in the initial distribution, and the constant perturbation from equilibrium states due to coupling clearly result in stronger peaks in the transient distribution. If the density approach can be extended to coupled maps (perhaps building on Wu's approach to produce an equivalent single map) it may be possible to more directly construct a classifier for a particular initial distribution characterizing some family of patterns to be recognized. For now, adaptive learning methods, such as I use in this thesis, seem to be the only practical approach.

SPATIALLY EXTENDED DYNAMICS, TRANSIENTS, AND SYNCHRONIZATION: NOTES ON THE LITERATURE

In this section I will survey additional literature exploring nonlinear oscillation dynamics in vision, pattern recognition and other engineering tasks, but which is somewhat tangential to the main thread of shape representation and similarity.

Conceptual Ties with Cellular Automata Literature

The fields of cellular automata (CA) and random boolean networks were the better established "parent" disciplines which spawned work on coupled maps (Wolfram 1986). Both share discrete time and space iteration, with most work employing synchronous update at all cells. CA and coupled maps systems differ only in that CA typically have boolean or small integer state variables, with boolean transition functions. Coupled maps use on or more real-valued numbers as state values, and use algebraic or piecewise-linear functions as update rules. *Transients* in cellular automata have been studied more extensively than in continuous dynamical systems or coupled maps; methods for creating appropriate structure of the attractor basins (i.e. the transients leading to an attractor) have been derived by Wuensche, but these computations have no obvious mapping to biological dynamics (Wuensche 1996).

The concept of *time-varying spatially extended dynamics* was proposed by Wolfram in the context of cellular automata (Wolfram 1986). A slow lattice controls the rules governing the update of sites on a fast lattice. Wolfram informally describes several strategies for pattern classification with such systems, covering some of the same ground described earlier by Rosenfeld (Rosenfeld 1979).

Pattern formation phenomena are the major object of study in CA, but are typically not considered as synchronization; however, results (reviewed in an earlier section) on problem solving strategies involving regular domains (Hordijk, Crutchfield et al. 1998), (Mitchell, Crutchfield et al. 1996) may be possible to recast in terms of synchronization. The essential difference is that *n-blocks* (words or spatial configurations) in CA play the role that *discrete phase space intervals in a single cell* play in coupled map lattices. Measures on block statistics and correlations replace measures on occupancy of phase space regions. Cooperative pattern formation processes are the essential characteristic in both systems.

Pattern Processing in Coupled Maps

Work on the processing of spatial patterns by arrays of chaotic units has been performed by Farhat and del Moral Hernandez (Farhat and del Moral Hernandez 1996). The standard symmetrical logistic map formulation is used for units, with the state variable interpreted as phase in the interval $[0,2\pi]$. They interpret the map as a model of spike processing in a single neuron, in contrast to a large scale network as in most other work reviewed in this section. One notable aspect of this work is that the coupling function between cells is itself nonlinear; two variants of coupling are proposed. One is an exponential function of input, the other a series quantization thresholds against this exponential function. Quantization (binning) results in a loss of smoothness in the characteristic pitchfork bifurcation diagram of the coupled maps, producing instead constant values until bifurcation points. The quantization is interpreted as different neurotransmitter release characteristics, associated with different presynaptic activation levels.

In their demonstration of pattern processing, piecewise linear activation values are applied as bifurcation parameters to an input logistic ring, which is coupled to a second processing layer via the nonlinear scheme above. Coupling between elements is homogenous. It is shown that after long transients (1700 cycles in one example) for some input patterns the dynamics may collapse to clusters of periodic attractors. The number of clusters is much smaller than the array, i.e. 6-7 clusters in an ring of 100 chaotic units. It is suggested that this convergence to clusters of periodic attractors for “coherent input” may be interpreted as recognition and classification of the input, while inputs which are not recognized remain incoherent.

As noted in my review, research in IT cortex, the putative site of object level feature recognition, has not turned up obvious periodic oscillatory dynamics fitting this hypothesis at the single neuron level, but neurons could be participating in larger scale aperiodic oscillatory dynamics. However, the large number of iterations required seems inconsistent with rapid processing. The correspondence of an iteration cycle with particular micro-circuit parameters is not developed in the paper, but even if it corresponds to recurrent processes in dendritic spike processing 1700 iterations seems a heavy burden to justify biologically.

A fully connected network of quadratic maps with period doubling route to chaos has been studied by Carvalho and coworkers (Carvalho, R. et al. 1999). A baseline bifurcation state at the critical transition to chaos is chosen, with a correlation learning rule based on the response to input. They note that uncoupled dynamics at this transition, while not chaotic in the + Lyapunov sense, consists of an infinite number of unstable periodic orbits; when coupling is induced between units, some of these are stabilized, resulting in a characteristic distribution over a set of high period orbits for the learned input pattern. This stabililization is observed for a fixed initial condition prior to presentation of patterns and is measured after 2×10^4 time steps.

Robert Gregson has, over the course of many years, explored nonlinear models of psychophysical phenomenon and collected the results in two monographs (Gregson 1988; Gregson 1995). Some of this work has utilized spatially extended or *field* models, designated as (nxn) Γ models. These are notable in the present context because many of the studies also deal with low numbers of iterations, thus are essentially exploring the computational correspondence of transient phenomena with psychophysical events. Also, Gregson introduces the notion of *cascades*, a set of n recursions in a lattice; the output of this system is fed back to the input for an "outer loop" of some number of iterations. In this model, the initial "stimulus" variables are not the state variables but rather gain values affecting the evolution of an autonomous complex variable; in the outer loop, the output of one such cascade is used to control the gain in a subsequent cascade. Gain in the n-Γ system serves as a bifurcation parameter, so the system as a whole is non-stationary and effectively *auto-bifurcating* in my own terminology introduced in a previous thesis (DeMaris 1995). That specific kinds of computations are effected by bifurcation changes *on a slow scale relative to evolution equations* is a major commonality with the present model [26]. Gregson has modeled spatial vision phenomenon such as the Muller-Lyer illusion, using total iteration counts under 100. He makes many points which I arrived at independently; that nonlinear evolution equations are a "total system analogue", rather than corresponding to any local (retinal or cortical) neural sheet. Also, in contrast to earlier field theories (Ratliff 1965), there is no reliance on opposed excitatory and inhibitory influences. Like the network dynamics explored here, Gregson notes that coupling connections between these nonlinear field units have no obvious interpretation as excitatory or inhibitory.

The phenomena of ambiguous perceptions has been of great interest since the earliest days of visual psychology. The spontaneous switching of the images such as the Necker cube is clearly a dynamical phenomena, and the apparent instabilities might be expected to shed light on perceptual processes. The literature on Necker cube psychophysics details interactions between eye movements and switching events, as well as interactions between scale, orientation, and the distribution of switching times. In an

[26] Because of this emphasis on transients, Gregson's approach must be acknowledged as a key precursor of my work, though the original impulses for my investigations came from other work outlined here; due to terminology differences I only realized the similarity of CML with his "cascades and fields" approach after personal communication with T. Henmi, comparing our respective work on Muller-Lyer illusions and attempting to combine aspects of both.

earlier study motivated by these complexities, I modeled the cooperative formation of monocular depth fields and attentional foci using a multi-layer CML model, mixing two locally coupled lattices with a globally coupled lattice. Scale changes in the cube led to changes Scale changes in the cube led to changes in the distribution shape conforming to psychophysical trends. A low dimensional dynamical model by Kelso et al. previously linked interactions between reversal rates and distribution shapes, but did not provide details of interactions between spatial forms and coupling, or account for attentional correlates (Kelso, Case et al. 1995).

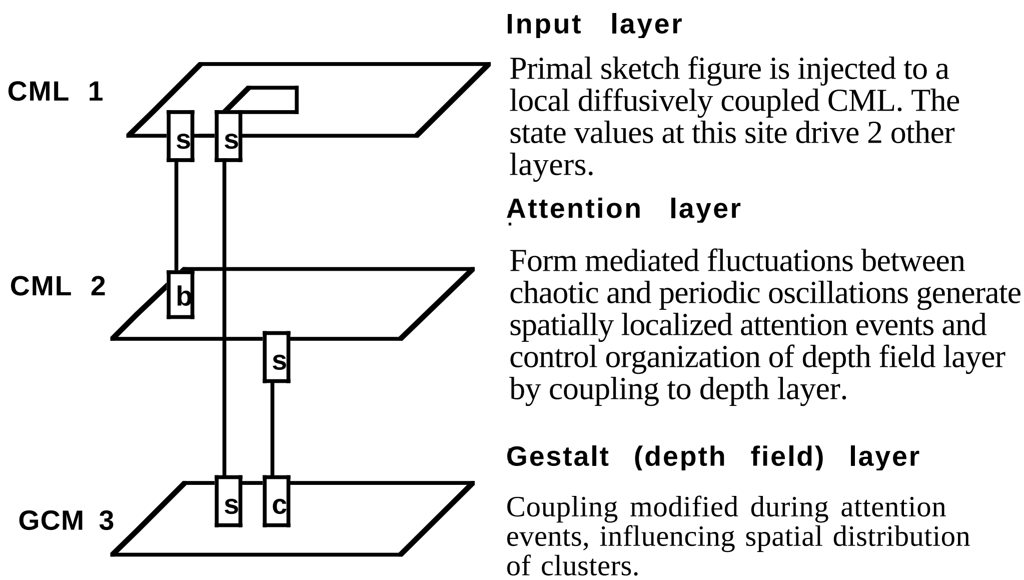

Fig. 21. Schematic of a network modeling formation of monocular depth fields with multiple CML layers. The labeling on the states indicate the nature of couplings between layers; i.e. state fluctations in layer 1 influence bifurcation parameter in layer 2. From DeMaris, D. (1998). Pattern formation in spatially extended nonlinear systems: toward a foundation for meaning in symbolic forms. !st. Intl. Conf. on Anticipatory Systems, Liege, American Institute of Physics. Reproduced with permission of Lawrence Erlbaum Associates.

Segmentation and Synchronization in Coupled Oscillator and Coupled Map Systems

Coupled oscillator and coupled map systems have by now been investigated as models of several perceptual tasks and phenomena. Segmentation, the task of identifying object boundaries and separating a scene into object tagged regions, has received the most attention. A few of these studies are relevant because of their intrinsic interest in the context of computing with synchronization processes, and also in that practical work in **computerized scene analysis** demands that several tasks be solved nearly simultaneously, such as the discrimination of object boundaries and the recognition of the objects demarcated. Successful segregation of objects is assumed in the computational work here, but in future engineering systems or more complete biological models these tasks may be combined; happily it appears that the synchronization framework, and chaotic synchronization in particular can support both tasks.

Peréz applied self-organizing responses at the local cell level in an inhomogeneous coupled map lattice model, using the evolution statistics of a "spin" order parameter as a discriminator of irregular structures in a regular background in a semiconductor defect recognition task (Perez 1988). By self organizing, I mean that the bifurcation parameter at each time step is a function of the state of each cell; this is further regulated by a local spin-correlation measure, which is essentially a measure of synchrony. Spin is defined as a difference measure from one time step to the next; positive increments are spin up, negative spin down. Thus neighboring cells with similar time series derivatives are stabilized, leading to segmented spatial activity domains corresponding to manufactured shapes regions and defects.

Price et al. (Price, Wambacq et al. 1993) used a coupled map lattice forced by damped sinusoidal modulation of a sample image for texture segregation, with results comparable to other approaches. Particular strengths claimed for the technique are relative insensitivity to the dynamic range and contrast of the original signal, avoiding the tuning usually associated with adaptive filter approaches, and the ability to effectively overcome the classic paradox of region segmentation in noise: how to smooth noise without blurring essential features. More recently, two groups have presented work on segmentation with continuous formulations coupled oscillator models. A system based on simplified Wilson-Cowan Oscillators with local spatial coupling has been developed by Campbell and Wang, with a particular emphasis on fast-synchronization, demonstrating that a network of several hundred oscillators in a one dimensional chain can be entrained in a single cycle (Campbell and Wang 1996). Coupling is dynamic, so that once groups are synchronized the coupling disappears. A special long range connection “global separator” unit acts on all local units to desynchronize oscillations by adjusting their parameters; this effect can be overcome by sufficiently strong local diffusive coupling, giving rise a sequence of activations of objects in connected regions. The state variables of the system are activity levels, with different objects represented by phase separation in time, with some clusters active while others are silent. Up to nine objects could be represented and separated by the system.

Another group has focused on overcoming what they term the “Synchrony - Desynchrony” dilemma, resulting from conflicting requirements for synchrony of oscillators coding the same object and desynchronization between clusters coding for different groups (Zhao, Macau et al. 2000). Like Campbell and Wang, they used Wilson-Cowan oscillators, but with Laplacian (2^{nd} derivative) coupling between oscillators, and with the parameters of the system such that *chaotic* oscillations result.

They claim that this allows effectively unlimited segmentation. It does so, however, by increasing the complexity of read-out of the encoded segmentation. Their procedure for identifying objects in the oscillating field involves observing which sets of oscillators visit a particular phase space region (Poincaré section) simultaneously. Since the chaotic trajectories of different clusters may coincidentally cross the section simultaneously, a decision on the assignment of groups is not made until 3-4 such simultaneous visits are made within a particular time interval. By that time, sensitive dependence is presumed to have separated the trajectories sufficiently.

Finally, I highlight another study using the coupled map lattice formalism to perform segmentation which goes further than most in seeking psychological plausibility by fitting psychophysical data on ambiguous perceptions (van Leeuwen, Styvers et al. 1997). As part of that project, a numerical study of C_{crit} , the critical coupling value leading to synchronization between two maps over a range of random initial conditions, against the bifurcation parameter was performed. The relation is not strictly monotonic, but does generally show increasing C_{crit} for increasing b parameter. C_{crit} Values in the range .16 to .25 appear in the chaotic bifurcation regime.

In the network of van Leeuwen et al., the presence of a signal in the input field reduces the bifurcation parameter of the map to the minimum of the specified range. The background state of the network is uncorrelated, chaotic oscillatory activity.

Coupling is adaptive to a smoothed difference measure between coupled nodes in this model, with weights scaled by a sigmoidal function of the difference function up to a maximum coupling. Therefore, spatiotemporal patterns of synchrony are achieved, with varying stability depending on the parameterization. With the addition of on axis directional preference to weight adjustments, switching between alternative Gestalt organizations is in evidence, with the distribution of switching times qualitatively matching psychological data. It is noted that this distribution is obtained with only one free parameter. A numerical study determined the critical coupling values leading to convergence over a range of random initial conditions vs. the bifurcation parameter. The relation is not strictly monotonic, but does generally show increasing C_{crit} for increasing b parameter. Values in the range .16 to .25 appear in the fully chaotic regime.

SUMMARY

The evolution of states in arrays of coupled discrete oscillators is a rich source of phenomena, ranging from attractors of various types, synchronization and clustering, complex transient structures, and spatial pattern formation. For the diffusive coupling and low iteration counts used in the present work, a few simple trends are evident. Increasing the coupling across a lattice decreases the effective dimension and nonlinearity; units which would be chaotic if uncoupled will become synchronized chaotic (for high b parameter) or even periodic with high coupling. The dynamics of response to structured inputs, used as either initial conditions or to modulate bifurcation or coupling parameters, is relatively unexplored.

Chaotic dynamics can be produced in small circuit neural models and larger ensembles through a variety of underlying pathways. Given the aperiodic, stimulus linked rate modulations and changes in correlation seen in biological networks, to study the dynamics of coupled chaotic systems seems a natural direction for neural modeling.

A researcher familiar only with the well known principles of *low dimensional chaos* – the resemblance of chaos to noise, the sensitivity to initial conditions - might dismiss the relevance of coupled recurrent chaotic systems as a model of neural processing. The added complexity of spatial interactions and coupling, however, can push a CML system either towards linearity (i.e. regarding the temporal or instantaneous statistical response to input), or may provide the substrate for very complex computations, such that correlations between input patterns and measures of the system response become useful tools for neural system design.

To date, very little research has gone beyond pure dynamics studies to perceptual modeling or pattern recognition with chaotic or periodic oscillatory systems. Segmentation is the most well studied area, and I have reviewed several recent contributions from other investigators.

In the next chapter I investigate the ability of oscillatory systems to rapidly form responses to spatial forms. I begin with the parametric study of transients in coupled logistic maps to spatial forms, and ultimately demonstrate a system for recognizing 3 dimensional objects from their 2 dimensional silhouettes. The demonstration shows that if the assumptions of place coding are abandoned, coupled map systems can serve as the physical substrate for algorithmic approaches ranging from the classical (e.g. metric spaces) to more modern (e.g. view based normalization).